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BCF-003-001513 Seat No. ________

B. Sc. (Sem. V) (CBCS) Examination

August – 2021

Mathematics : BSMT - 501 (A)

(Mathematical Analysis - I and Group Theory)

(Old Course)

Faculty Code : 003

Subject Code : 001513

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions :

1) All the questions are compulsory.

2) Numbers written to the right side indicates full marks of the question.

1. Answer all the Questions **[20]**

1) Define Riemann Integration.
2) State Darboux's Theorem.
3) Define Refinement.
4) Define Oscillatory Sum.
5) State Second Mean Value Theorem for Integration of Bonnett's Form.
6) If $P = \{1, 7.5, 15.5, 20\}$ is partition of $[1,20]$ then find $\| P \| =$ _____.
7) Define Interior Point.
8) State Hausdorff's Principle.
9) Define Discrete Metric Space.
10) Define Dense Set.
11) Give an example of neither open nor closed set in standard metric space.
12) Define Non–Singular element.
13) Define Abelian Group.
14) Define Idempotent element.
15) Define Coset.
16) Find the order of element 4 of $(\mathbb{Z}_6, +_6)$ and also find total number of generators of $(\mathbb{Z}_6, +_6)$.
17) Give an example of non–cyclic group which is an abelian.
18) Define Quotient group.
19) Define Isomorphism of Group.
20) Define order of $\mu = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 5 & 3 & 4 & 2 & 1 & 8 & 7 & 6 \end{pmatrix} \in S_8$

2(a). Answer any THREE **[06]**

1) Prove that $\int_{\underline{a}}^{b} f(x)dx \leq \int_{a}^{\bar{b}} f(x)dx$
2) If function f defined as : $f(x) = 0 \,; x \in \mathbb{Q}$
   $= 1 \,; x \notin \mathbb{Q}$ Then Show that
   f is not R–Integral over $[a, b]$.
3) If f is continuous in $[a, b]$ then prove that $f \in R_{[a,b]}$.
4) Prove that finite intersection of finite open set of metric space is open Set.
5) Prove that $\mathbb{N}' = \emptyset$.
6) If (X, d) is Metric Space and $A, B \subset X$ then prove that $A \subset B \Rightarrow A' \subset B'$.

(b). Answer any THREE **[09]**

1) State and Prove First Mean Value Theorem for R–Integration.
2) State and Prove Fundamental Theorem of Integration.
3) Evaluate : $\lim_{n\to\infty} n \sum_{r=0}^{n-1} \frac{1}{n^2+r^2} = \frac{\pi}{4}$
4) If $f, g \in R_{[a,b]}$ then prove that $f + g \in R_{[a,b]}$
5) In usual notation prove that $(\mathbb{R}, d)$ is metric space.
6) Prove that every convergent sequence is Cauchy Sequence.

(c). Answer any TWO **[10]**

1) State and Prove Necessary and Sufficient Condition for a bounded function f on $[a, b]$ to be R–Integrable
2) For $0 < x < \frac{\pi}{2}$, Show that $f(x) = cos(x)$ is R–Integrable and Find $\int_0^{\frac{\pi}{2}} \cos(x)\, dx$.
3) State and Prove General form of First Mean Value Theorem.
4) In usual notation prove that $\overline{E}$ is closed set.
5) If (X, d) is metric space, then show that $\left(X, \frac{d}{1+d}\right)$ is also metric space.

3(a). Answer any THREE **[06]**

1) State and Prove Reversal law for a group.
2) Show that Inverse element is unique.
3) Prove that every element of a finite group is a finite order.
4) Prove that a group G is commutative if $(ab)^2 = a^2b^2, \forall\, a, b \in G$.
5) Check whether given permutation even or odd and also find $O(f)$ where
   $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 3 & 5 & 2 & 8 & 6 & 1 & 7 \end{pmatrix} \in S_8$
6) If G is a finite group then show that $a^{O(G)} = e \,, \forall a \in G$.

(b). Answer any THREE **[09]**

1) Show that a non–empty subset H of Group G is subgroup of G iff $ab^{-1} \in H$.
2) Let G be a group and $a, b \in G$ such that $a \neq e$ & $O(b) = 2$, If $bab^{-1} = a^2$ then find $O(a)$.
3) Write all the element of S_3 also find order of each elements of S_3.
4) Define Translation, Invariant and Transposition with example.
5) Let $H \leq G$ and $K \leq G$ then Prove that $K \cap H$ is normal subgroup of K if H is normal subgroup of G.
6) Prove that any two disjoint cycle in S_n is commutative.

(c). Answer any <u>TWO</u> **[10]**

1) State and Prove Lagrange's Theorem.
2) State and Prove Cayley's Theorem.
3) A subgroup H of group G is normal subgroup iff $(H_a)(H_b) = H_{ab}$; $\forall a, b \in G.$
4) Show that the mapping $f : (\mathbb{R}, +) \rightarrow (\mathbb{R}^{+}, \times)$ is defined by $f(x) = e^{x}$; $\forall x \in \mathbb{R}$ is an isomorphism.
5) Define Alternating group A_n, Show that $A_n(n \geq 2)$ is subgroup of S_n of order $\frac{n!}{2}$.
